CRYSTAL
INSTRUMENTS

Dynamic Stiffness Measurement or Conversion

Application Note 082

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June 2022 |

Contents

Discussion of Dynamic Stiffness

Frequency Response Function (FRF) measurement is widely used in structural testing. Accelerometers are typically used and the acceleration vs. excitation force is measured as FRFs.

In case of displacement vs. force, the FRF equation is described as the following:

$H(f) = X(f) / F(f)$

where,

X(f), displacement spectrum
F(f), force spectrum
H(f), FRF

Since acceleration, velocity and displacement can be involved, three types of FRF functions are available. Considering the inverse FRF, there will be another three types.

The inverse FRF equation is described as the following:

$H(f)^{(-1)} = F(f) / X(f)$

where,

F(f), force spectrum
X(f), displacement spectrum
$H(f)^{(-1)}$, inverse FRF

The Table 1 lists all these FRF and inverse FRF types.

The most common type is *Accelerance*, which is the response in acceleration vs. excitation force. If the response is *Velocity*, the ratio is *Mobility*. The response of displacement vs. excitation force is *Compliance*.

The inverse FRFs are *Dynamic Mass*, which is force vs. acceleration. The velocity response is *Mechanical Impedance*. When force vs. displacement is calculated, the result

	Acceleration	Velocity	Displacement
(FRF)	Accelerance	Mobility	Compliance
Output/ Input	Acceleration/Force	Velocity/Force	Displacement/Force
(Inverse FRF)	Dynamic Mass	Mechanical Impedance	Dynamic Stiffness
Input/ Output	Force/Acceleration	Force/Velocity	Force/Displacement

Table 1

Figure 1.1

is *Dynamic Stiffness*,

Dynamic Stiffness is clearly the inverse of *Compliance*.

Accelerometers are widely available and inexpensive to acquire. On the other hand, displacement sensors are more expensive when considering the laser type. The frequency range of a laser displacement sensor is also limited compared to accelerometers. Typical laser displacement sensors support a frequency range up to 300 Hz and high-end laser displacement sensors support a frequency range up to 1 kHz. On the other hand, accelerometers can easily cover to 10 kHz or an even higher range.

The preceding information confirms the many advantages of using accelerometers. Users can derive other types of FRF measurements from acceleration by converting acceleration to velocity or displacement, or vice versa. Crystal Instruments data acquisition system supports this conversion. This feature allows users to obtain values such as dynamic stiffness from the acceleration measurement.

Test Setup

The Spider-80X data acquisition system and EDM DSA software were used to conduct the FRF measurement. The goal was to verify the FRF calculation and the conversion of other types.

The photo in Figure 1.1 shows a plate suspended with bungee cords.

A smart shaker was used and white noise up to 720 Hz was generated to excite the plate under test. A force sensor was placed on the excitation point connected to the shaker through a stinger.

An accelerometer was placed on one side of the plate. The laser displacement was placed with

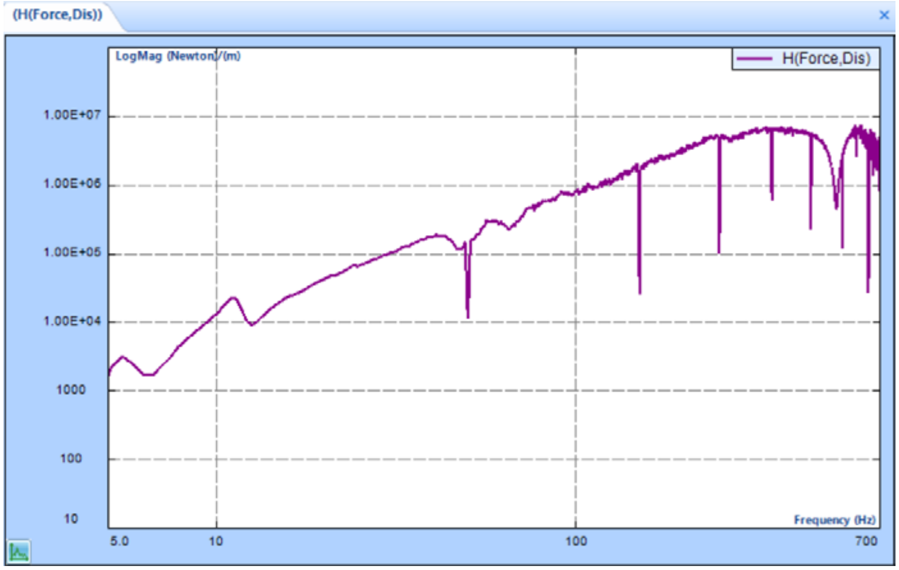

Figure 1.2

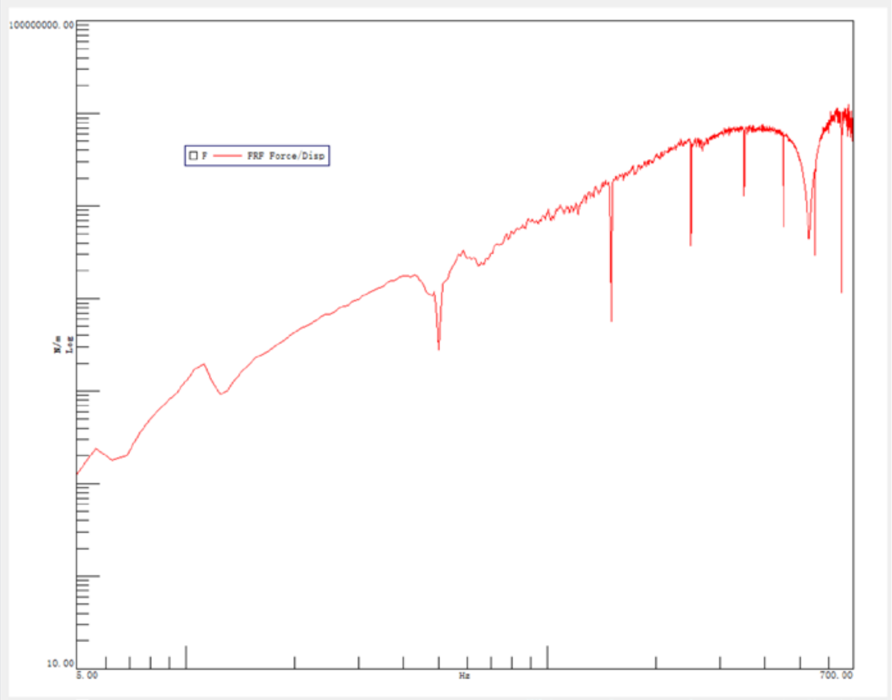

Figure 1.3

a defined distance close to the accelerometer. These locations are considered the same.

Data was collected and analyzed using the three sensors listed in the test conditions. FRF and inverse FRF are calculated using Both EDM DSA and another dynamic signal analyzer.

Measurement from both instruments verified the FRF measurement as well as the inverse FRF measurement. Also, the conversion of dynamic mass to dynamic stiffness was verified against a direct measurement.
The sensor sensitivities are listed as follows:

- Force sensor, 11.241 mv/N
- Laser displacement sensor, 400 mv/mm
- Accelerometer, 10 mv/g

Test Results

1. Dynamic stiffness of direct acquisition (Figure 1.2 and Figure 1.3)

Dynamic stiffness is directly measured using the laser displacement sensor and the force sensor. The following two dynamic stiffness signals are measured using EDM DSA and another dynamic signal analyzer.

* Horizontal axis vertical axis log-log, horizontal axis 5 – 700 Hz, vertical axis unit N/m

2. Compliance for direct acquisition

The FRF of Compliance is measured directly from the laser displacement sensor and force sensor. The following screenshot displays Compliance signals measured from EDM DSA and another dynamic signal analyzer. (Figure 1.4 and Figure 1.5)

* Horizontal axis vertical axis log

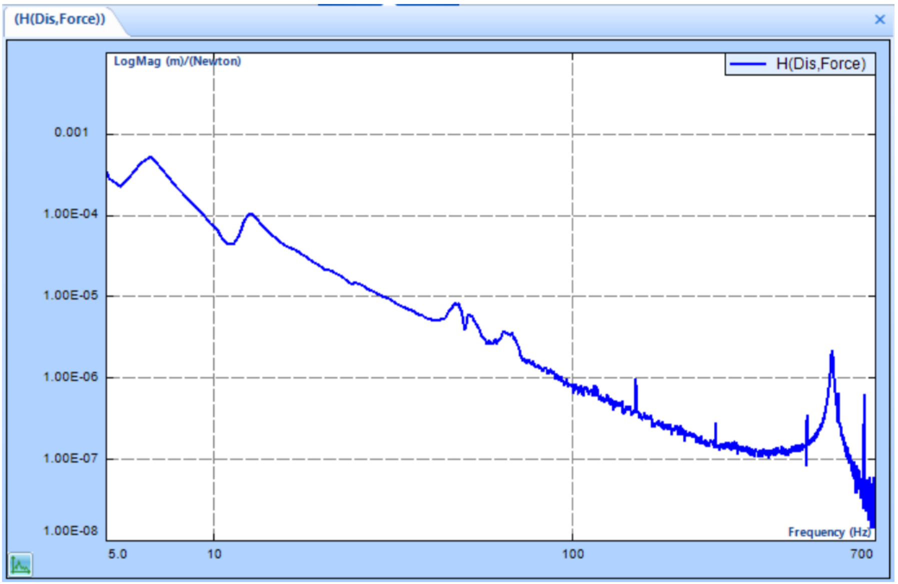

Figure 1.4

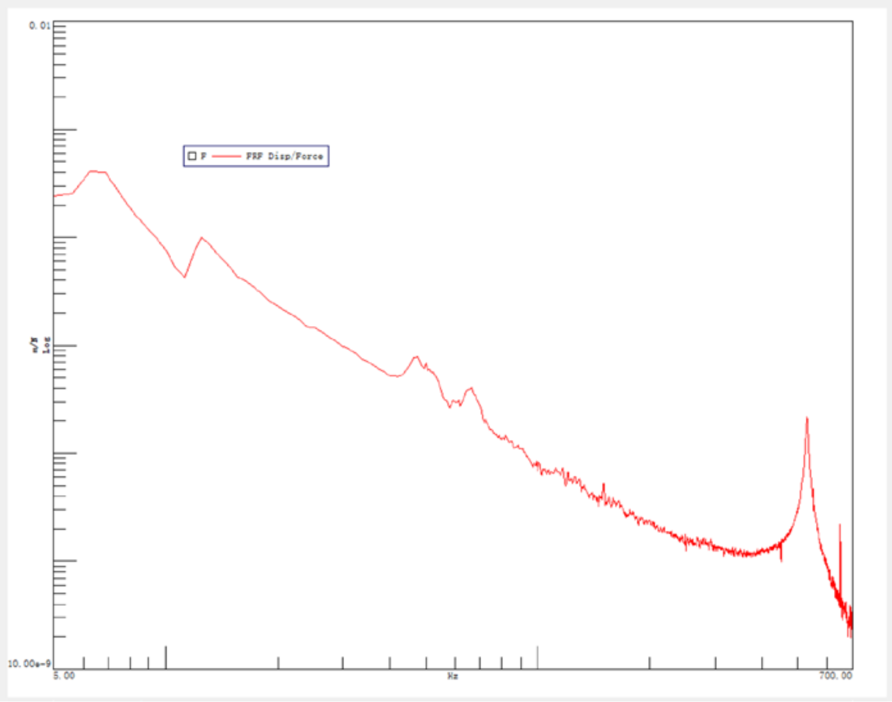

Figure 1.5

display, horizontal axis 5 – 700 Hz, vertical axis unit m/N

3. Accelerance for direct acquisition

The FRF of Accelerance is measured directly from the accelerometer and force sensor. The following screenshots show the Accelerance signals measured from EDM DSA and another dynamic signal analyzer. (Figure 1.6 and Figure 1.7)

* Horizontal axis vertical axis log display, horizontal axis 5 – 700 Hz, vertical axis unit g/N

4. Dynamic stiffness (red curve) and dynamic stiffness after switching of dynamic mass AVD (green curve)

The following signals show Dynamic Stiffness measured directly from the displacement sensor and derived from the dynamic mass (force vs. acceleration). EDM DSA provides the feature of AVD (acceleration, velocity, and displacement) conversion for related signals. Any measured unit with the other two units and related signals can be converted directly within the EDM DSA graph display.

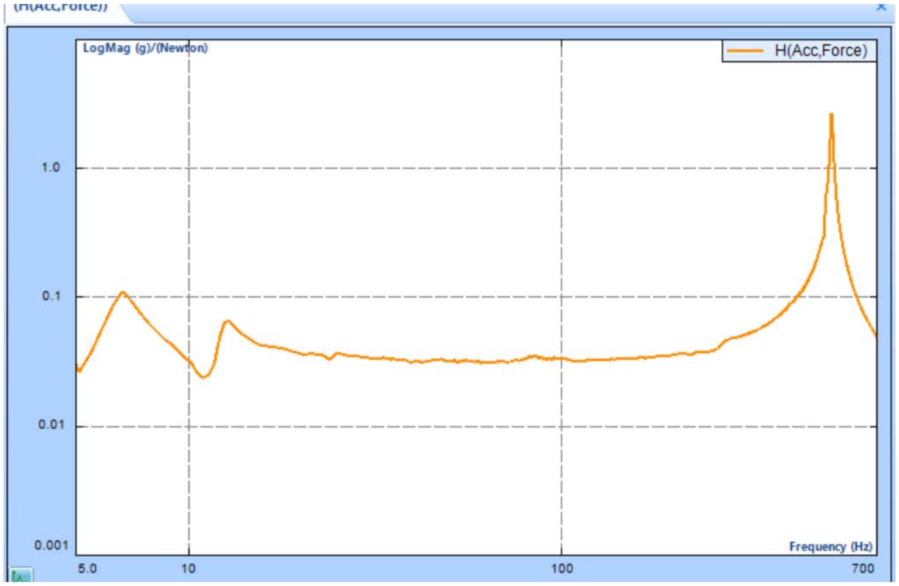

Figure 1.6

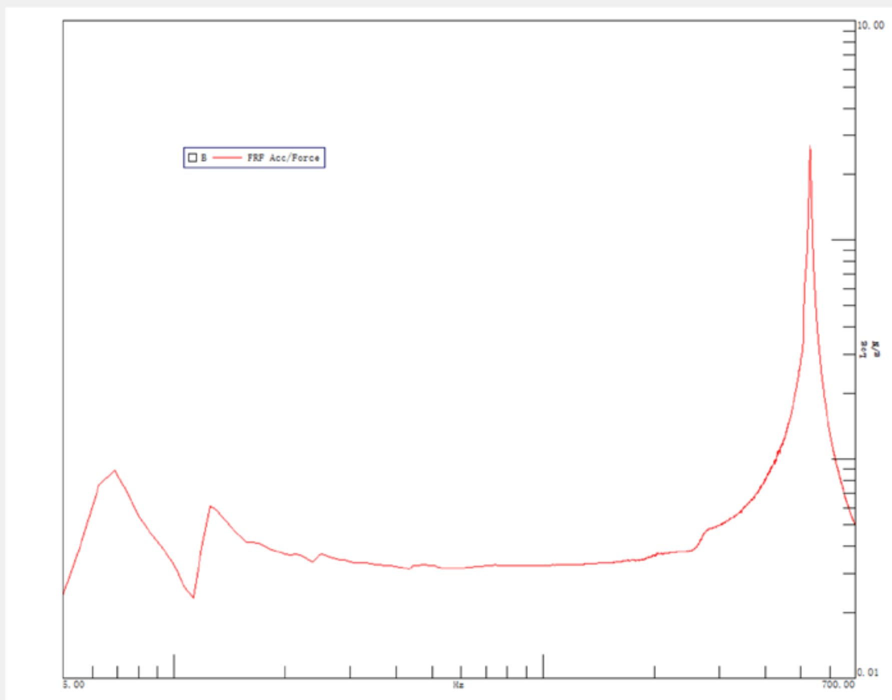

Figure 1.7

The graph in Figure 1.8 shows the direct measured Dynamic Stiffness from the laser displacement sensor (in red).

The next graph shows the dynamic stiffness (in green) converted from the dynamic mass (measured from the accelerometer). (Figure 1.9)

The preceding two dynamic stiffness signals obviously match each other. This verifies the functionality of EDM DSA AVD conversion. Additionally, users are provided the choice to use accelerometers and obtain displacement related signals such as dynamic stiffness.

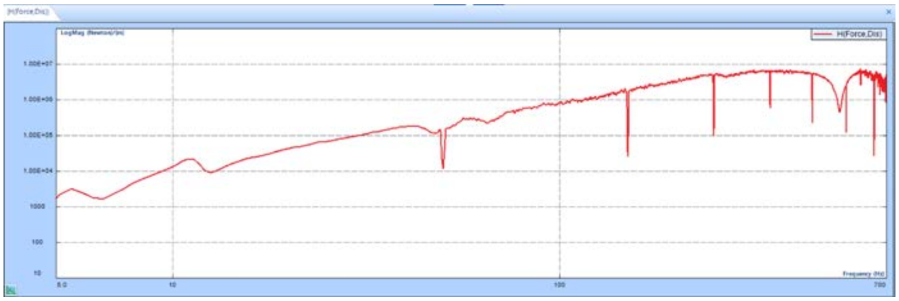

Figure 1.8

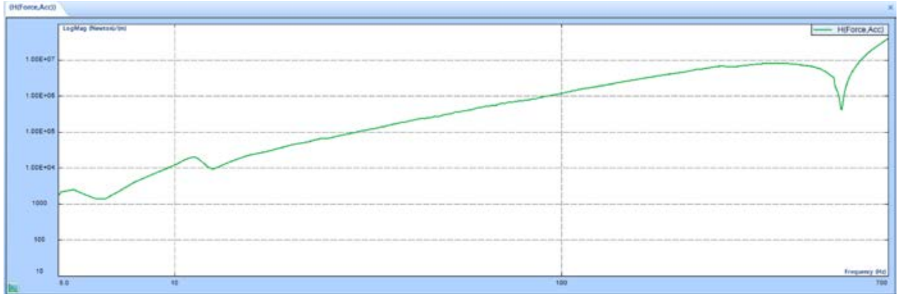

Figure 1.9