Fatigue Damage Spectrum

Application Note 089

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Fatigue Damage Spectrum

The Fatigue Damage Spectrum (FDS) serves as a pivotal tool in the assessment of structural integrity, portraying the relationship between damage and natural frequency across the frequency domain. Its utility extends to testing laboratories, empowering them to craft precise Random vibration test profiles by analyzing acceleration vibration data derived from real-world environments. By using the FDS, these labs can accurately delineate the damage experienced by structures under dynamic loads, facilitating the development of robust testing protocols essential for ensuring product reliability and safety.

The overall workflow is as follows:

1. For each vibration test recording, calculate the respective FDS plot.
2. Sum the corresponding FDS plot w.r.t each axis for a final FDS calculation.
3. Derive a final Power Spectral Density (PSD) and test duration for an equivalent fatigue damage.

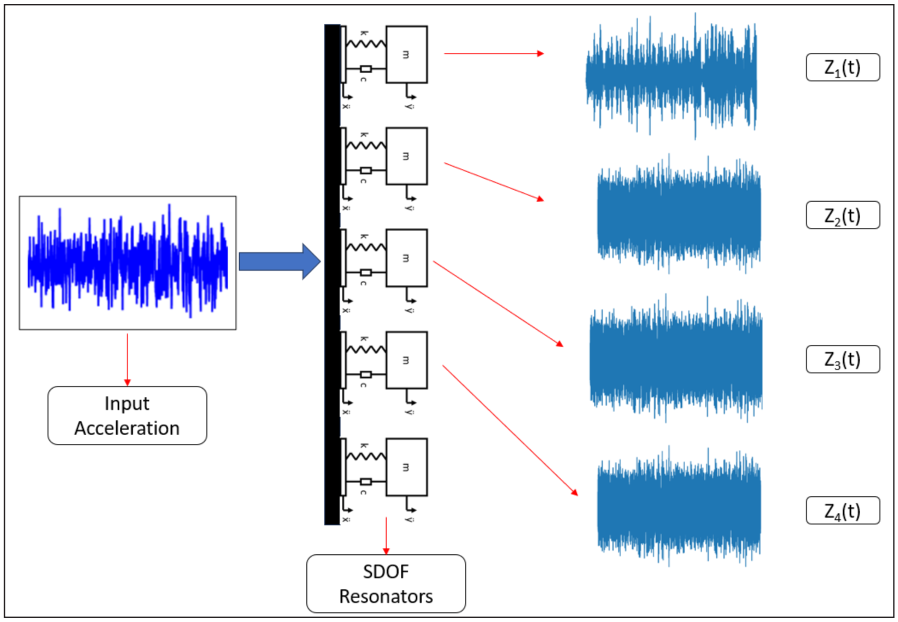

Figure 1: Base excitation provided to each SDOF

Calculating FDS from the time domain

The methodology employed in calculating the Fatigue Damage Spectrum (FDS) for acceleration data in the time domain parallels the approach used in deriving a Shock Response Spectrum (SRS) for a time domain shock pulse. In both instances, the process hinges on determining the Single-Degree-of-Freedom (SDOF) response to the input signal.

The Single-Degree-of-Freedom (SDOF) response is characterized by two critical parameters: damping and natural frequency. In the calculation of the Fatigue Damage Spectrum (FDS), damping is typically held constant at a predetermined value, often around 5%, although the user does have the discretion to edit this parameter. The SDOF response is subsequently computed across a spectrum of natural frequencies which are then represented as octaves. This meticulous process ensures a comprehensive understanding of how structures react to varying vibrational frequencies, laying the groundwork for precise fatigue damage assessment and mitigation strategies.

In practice, the Smallwood Ramp-Invariant Filter is applied to approximate the SDOF response. This Smallwood filter is a well-known IIR digital filter whose coefficients are calculated from the given sampling rate, damping, and natural frequency values. For best results, the underlying time domain excitation should be resampled to 10 times the natural frequency.

While calculating FDS from temporal data, the input acceleration is provided to each Single Degree of Freedom (SDOF) system, with the corresponding displacements determined through the application of the Smallwood Ramp-Invariant filter. Each SDOF system possesses a unique natural frequency, typically given in terms of octaves, which the user inputs into the algorithm. (Figure 1)

Once the relative displacements are obtained, the RMS of each response is calculated and stored in a variable named z_{rms}. The fatigue experienced by each SDOF due to the input acceleration is calculated as follows:

$$Fatigue = f_n * t_{life} * (\sqrt{2} * z_{rms})^m * \Gamma(1 + m/2)$$

where,

f_n	Natural frequency f_n of an SDOF
m	Damage Exponent
z_{rms}	RMS of SDOF response subject to input acceleration
t_{life}	Lifetime duration
Γ(g)	Gamma function
m	Fatigue exponent

The above equation is used to calculate Fatigue (damage) across the entire spectrum, which is the octave range given by the user. This plot of fatigue damage versus natural frequency is the final Fatigue Damage Spectrum. An example of this is shown in Figure 2, which shows the total damage accumulated by each SDOF.

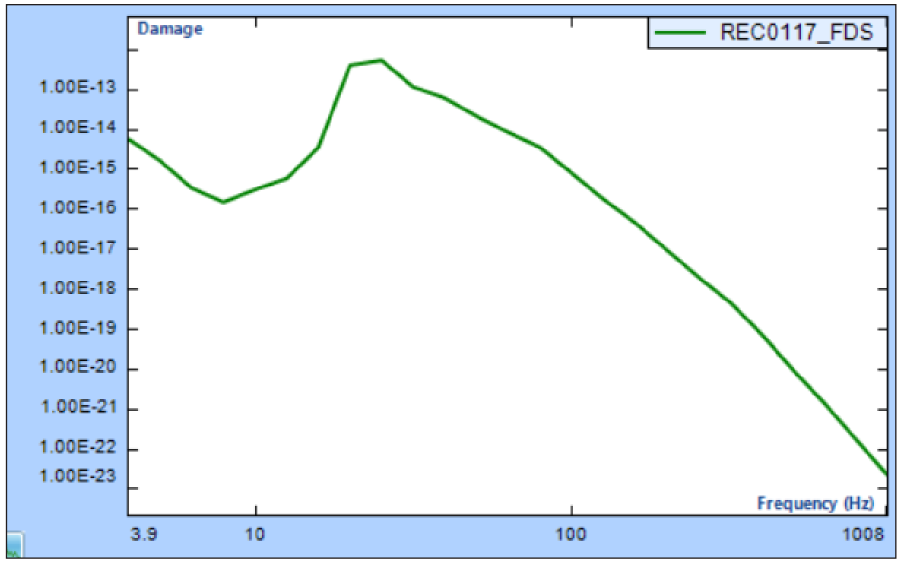

Figure 2: Plot of Fatigue Damage Spectrum

Calculating Accelerated Power Spectral Density (PSD) from an FDS

It is more straightforward to relate the frequency domain acceleration spectrum (abbreviated as PSD for Power Spectral Density) to the FDS. The following approximation can be used:

The variables below are defined:

$$\mathrm{PSD}(f_n) = \frac{2(2\pi \cdot f_n)^3}{Q} \cdot \left[\frac{D(f_n)}{f_n \cdot T_{\mathrm{test}} \cdot \Gamma(1+m/2)}\right]^{2/m}$$

$D(f_n)$	Fatigue Damage at frequency f_n
$PSD(f_n)$	Acceleration spectrum (EU^2/Hz) at frequency f_n
$Q=1/2\xi$	Q factor, expressed in terms of damping coefficient ξ
T_{test}	Test duration (how long the PSD is run on the shaker)
$\Gamma(g)$	Gamma function
m	Fatigue exponent

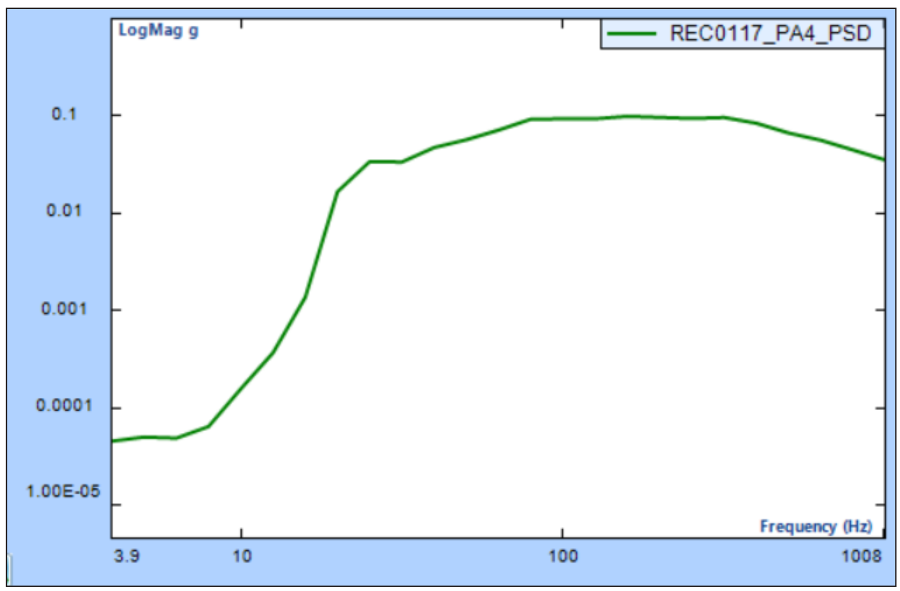

Figure 3: Accelerated PSD for a given Test Duration

The software calculates an accelerated PSD based on the given inputs to the above equation and plots it w.r.t to octave spectrum given by the user. An example of the resulting graph is shown in Figure 3.

The purpose of an accelerated PSD is that it yields the same damage that the DUT (Device Under Test) experiences through its "lifetime duration". The term accelerated denotes the fact that the testing time of the DUT is reduced but continues to account for the total damage. This is done so that the user doesn't have to run the shaker for hours (or the actual "lifetime duration"), which can be an expensive task.

Post Analyzer also accommodates situations where the user has several measured events such as "Takeoff", "Cruise" or "Landing". The FDS from each event can be summed up and an accelerated PSD can be calculated. However, care must be taken to sum FDS from the respective axes as well. For example, while using a tri-axial accelerometer for each of the above-mentioned events, each event will have measurements from X, Y and Z axes. The user will need to sum FDS from its corresponding axes only, i.e. X-axis from takeoff + X-axis from Cruise + X-axis from Landing. An accelerated PSD can then be obtained for just the X-axis. This enhancement will be available for users in the upcoming release.

Applications

To understand the benefits of FDS and an accelerated PSD more effectively, let's consider a simple scenario. Consider a car prototype undergoing a 15-minute road test. Now, imagine installing an accelerometer on the car to measure and record vibrations as shown in Figure 4. This setup provides valuable data on the car's vibration levels during the test.

After obtaining this recording, the user would likely aim to subject various prototypes to similar conditions, if not all types. However, simulating real-world conditions for every prototype isn't feasible. This becomes especially impractical when dealing with lengthy time streams spanning hours or days. In these scenarios, it would be beneficial to have a Power Spectral Density that can replicate such time stream recordings. The Fatigue Damage Spectrum application can accomplish this in the following steps.

1. Obtain the Fatigue Damage caused on the protype by using the equations mentioned previously in this paper. (Figure 5)
2. Back-calculate a PSD to run the same vibration levels on a shaker in a test lab. This PSD will represent a test time which is equal to the lifetime duration (a.k.a recording duration) which is 15 minutes. Notice that the RMS from the PSD in Figure 6 will be identical to the RMS of the timestream. This is a good way to ensure that the vibration energy levels are maintained between the real-world scenario and a laboratory test.

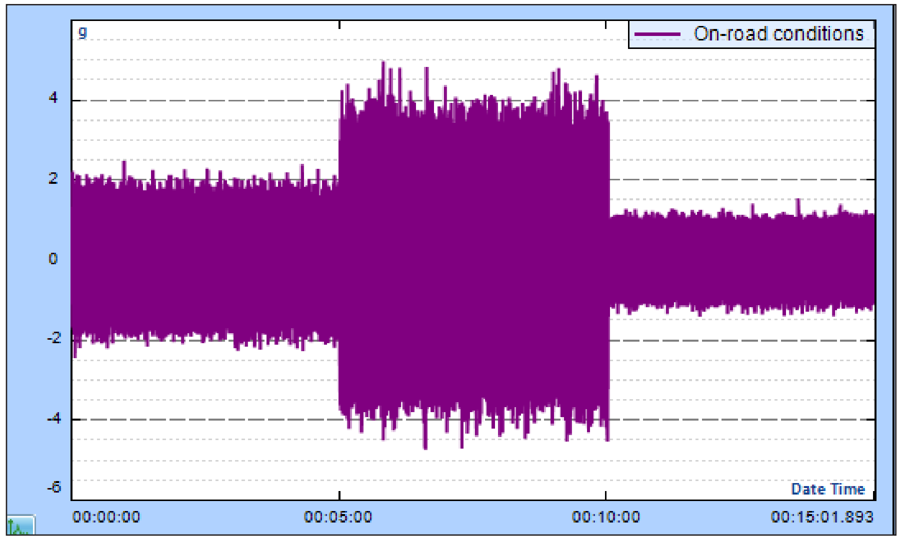

Figure 4: Time steam recording from an accelerometer

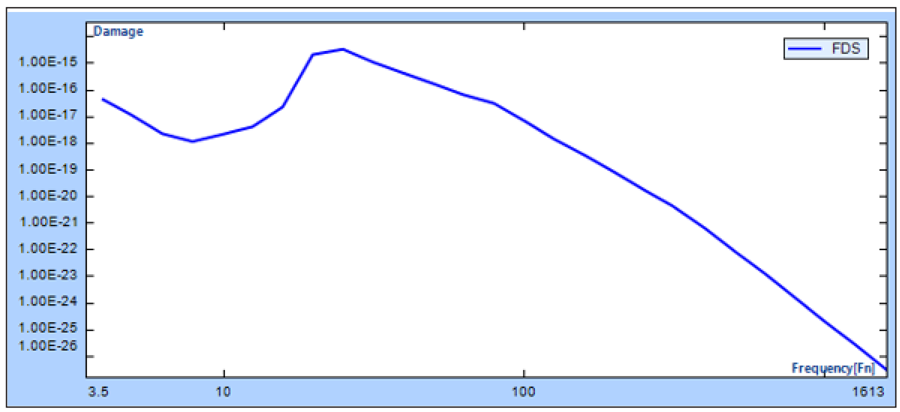

Figure 5: FDS obtained from the time stream.

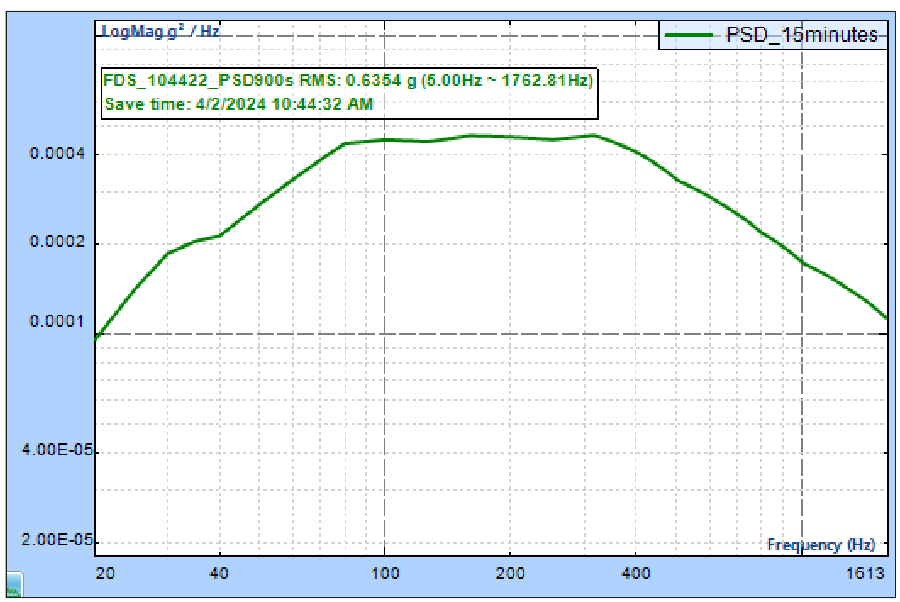

Figure 6: PSD for a test duration of 15mins.

3. To speed up testing time on a shaker, using FDS can help obtain an "accelerated" PSD. This results in causing the same fatigue damage to the prototype but in a much shorter time. This usually results in an increase in the Root Mean Square (RMS) of the test. For example, in Figure 7, the RMS of the accelerated PSD is 1.01.
4. The accumulated damage resulting from an accelerated PSD and the initial on-road testing can be compared for analysis. Figure 8 illustrates this comparison, showcasing the credibility of the FDS methodology presented in this paper.

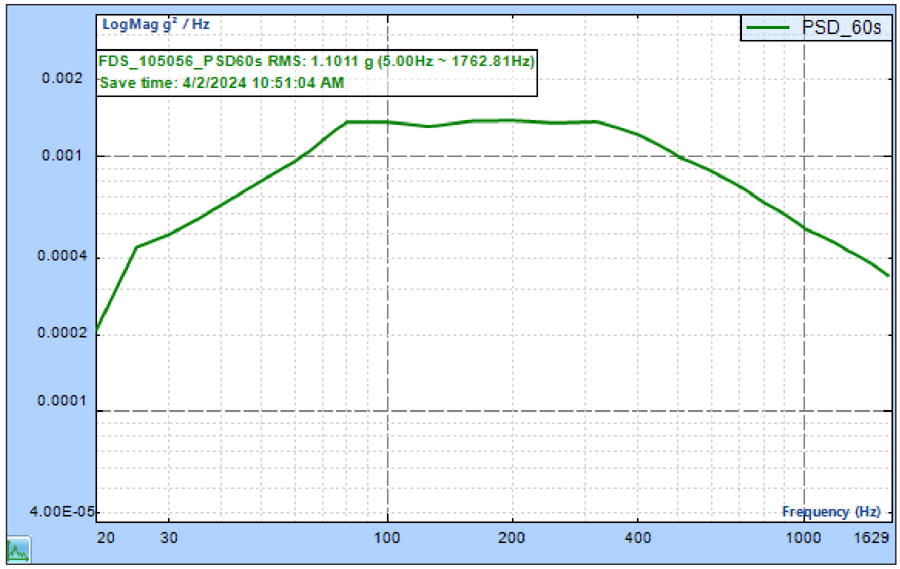

Figure 7: Accelerated PSD for a test duration of 1 minute

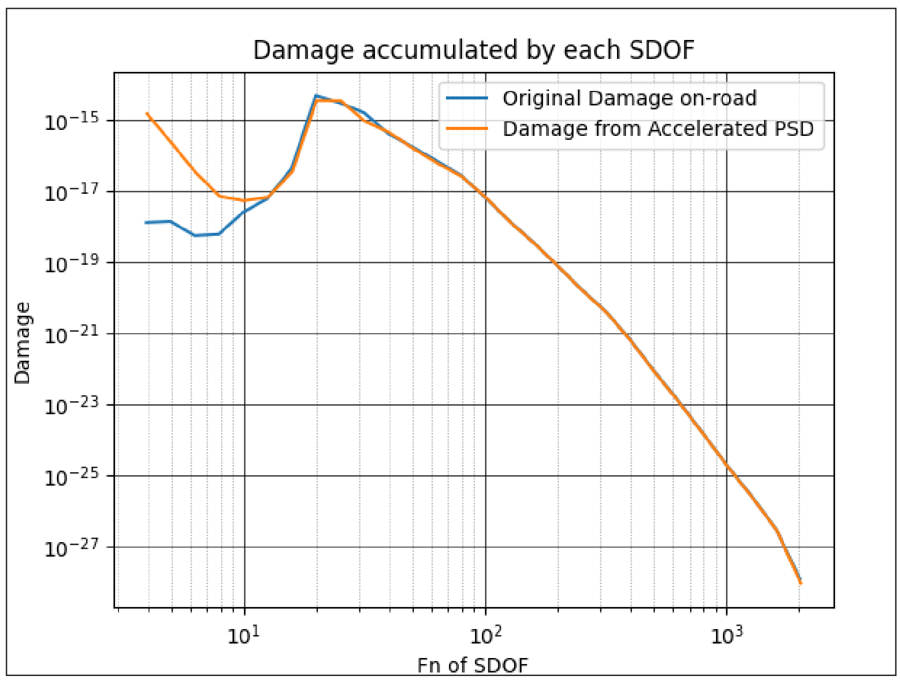

Figure 8: FDS comparison from on-road testing vs accelerated PSD

Conclusions

FDS offers a valuable tool for design engineers. By analyzing the frequency content of a test, it pinpoints the frequencies most susceptible to damage. This information can then be used to avoid those critical frequencies during the design phase of a structure. Additionally, FDS helps create more efficient testing profiles for shaker systems. This reduces overall testing time and associated costs.

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